Bivalence, Classical Logic and the Problem of Contingent Statements

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Abstract

The main objective of this paper is to argue that the principle of bivalence is right, contrary to the view of some philosophers. To fulfil this objective, the paper examined some arguments raised in Philosophy of Logic about the principle of bivalence starting from Aristotle's challenge to the principle of bivalence based on the idea that the principle cannot accommodate contingent statements. The paper examined Lukasiewicz's challenge of the principle of bivalence and Lesniewski's response to him. The paper evaluated these debates, in Philosophy of Logic, to determine if the principle of bivalence should be rejected. The paper employed the methods of logic. The study showed that the principle of bivalence had been misunderstood by some of the most influential proponents of many-valued logic, for example, Lukasiewicz. It was established that the terms true (or false), in the arguments against bivalence, was used in an epistemic sense and not a logical sense. It was established that contrary to Aristotle's and Lukasiewicz's assumption, contingent statements were necessarily either true or false; hence, the principle of bivalence could accommodate contingent statements. The paper concluded that the principle of bivalence is not in any way limited; it is the core of logic; Furthermore, there may be no conflict between the principle of bivalence and other systems of logic that are not strictly bivalent, if their justification does not rely on a rejection of the principle of bivalence.

Key words: Bivalence, Contingent statements, Classical logic, Non-contradiction, Necessity, Ontology

Introduction

Classical logic is the traditional form of logic and probably the most popular form. Classical logic relies on the principle of bivalence that states that every statement is necessarily either true or false. The principle of bivalence restricts the possible truth-values of statements strictly to 'true' and 'false'. However, Aristotle argued that this principle is not applicable to contingent statements; otherwise we will run into the problem of fatalism. Though Aristotle never created an alternative to classical logic, his essay provoked some philosophers into rejecting the principle of bivalence and creating alternatives to logic.

Classical Logic

Classical logic is a form of formal logic and probably the most popular and most studied form of logic. Some of the fundamental or underlying principles of classical logic are the three basic laws of thought credited to Aristotle. These three laws of thought are: the laws of excluded middle, non-contradiction and identity.[1]

Classical logic is also based on the principle of bivalence. The principle of bivalence states that there are only two values: true and false, and there is no room for any other value. There is hardly any controversy in logic about the principle of bivalence. However, there are controversies about whether it is true or not. We cannot meaningfully study classical logic without reference to the principle of bivalence. For a long time, classical logic was the only recognised form of logic, till when some questions arose as a result of Aristotle's essay on future contingents.

Law of Excluded Middle

The law of excluded middle is one of the traditional laws of thought credited to Aristotle. The law states that every statement is either true or not true. The law of excluded middle does not eliminate the possibility of having more truth-values apart from true and false. The law only asserts that for every statement P, P is true or P is not true. For example, if P represents the statement "All triangles have three angles", the logical disjunction "All triangles have three angles or it is not the case that all triangles have three angles" is true. The law of

excluded middle in symbolic logic is (Pv~P) is true. The definition of the law of excluded middle can be captured simply in Quine's words, "Schematically, the law of excluded middle is simply 'p or ~p'."[2]

The Principle of Bivalence

The principle of bivalence is one of the fundamental foundations of classical logic. The principle of bivalence states that every statement takes exactly one of these two truth-values: true and false. For any statement P, P is necessarily either true or false. The truth-value of P cannot be any other value apart from true or false. It is not only true that the statement "It is raining now" is either true or not true; the principle of bivalence goes further to say that the statement is necessarily either true or false.

Though related to the law of excluded middle, the principle of bivalence and the law of excluded middle do not assert the same thing. The law of excluded middle asserts that (Pv~P) while the principle of bivalence asserts that necessarily P is true or P is false. This is why some systems of many-valued logic specifically reject the principle of bivalence, but is still compatible with the law of excluded middle. There is hardly any disagreement on what the principle of bivalence asserts, but there are controversies on its truth. One of the most popular challenges to the principle of bivalence is Aristotle's essay on the problem of future contingents.

The Sea Battle Tomorrow

> A sea-fight must either take place to-morrow or not, but it is not necessary that it should take place to-morrow, neither is it necessary that it should not take place, yet it is necessary that it should or should not take place to-morrow.[3]

These words of Aristotle state the problem of future contingents that prompted some philosophers into creating alternatives to the logic of bivalence.

According to the proponents of classical logic, and Aristotle himself, the principle of bivalence is necessarily true and immutable; that is, it is binding and cannot be altered. However, Aristotle wrote an essay

that created a doubt about the truth of the principle of bivalence. Aristotle, however, was not the first to discuss the problem of future contingents.[4] But it seems that Aristotle's essay gave the problem modern recognition. Though there are different interpretations of Aristotle's argument in his essay on the problem of future contingents, I will give an interpretation which I think is a plausible interpretation of Aristotle's argument on the problem of future contingents.

Aristotle never mentioned "bivalence" in his works. However, we can infer that he clearly had an understanding of this principle from his works:

> For if, every affirmation or negation is true or false it is necessary for everything either to be the case or not the case. For if one person says that something will be and another denies this same thing, it is clearly necessary for one of them to be saying what is true – if every affirmation is true or false; for both will not be the case together under such circumstances.[5]

For Aristotle, it is a given fact that any declarative sentence should be necessarily either true or false. Aristotle also assumed that it is given that for any pair of contradictories, a statement and its negation, to be necessarily either true or false.

> Thus, if it is true to say that a thing is white, it must necessarily be white; if the reverse proposition is true, it will of necessity not be white. Again, if it is white, the proposition stating that it is white was true; if it is not white, the proposition to the opposite effect was true. And if it is not white, the man who states that it is making a false statement; and if the man who states that it is white is making a false statement, it follows that it is not white. It may therefore be argued that it is necessary that affirmations or denials must be either true or false.[6]

However, when we apply the principle of bivalence on statements that refer to future events, we run into the problem of future contingents. If all statements are necessarily either true or false, then the statement

"There will be a sea battle tomorrow" must be necessarily either true or false. A pair of contradictories is such that when one of the pair is true, the other must be false. That is, when one of this pair of contradictories "There will be a sea battle tomorrow" and "There will be no sea battle tomorrow" is true then the other is false.

If the truth-value of a statement referring to a future event is determined as either true or false, prior to the actual occurrence of that event, the implication is that the event has been predetermined. We run into the problem of fatalism. Fatalism assumes that events are predetermined and their occurrences cannot be changed; that is, what will happen, will happen no matter what. If we can tell if a contingent statement is true or false, it follows that we can determine future events prior to their occurrence; then, fatalism is true.

However, Aristotle argued that events are not predetermined. To avoid the problem of fatalism, we have to assume that contingent statements cannot be necessarily either true or false. To determine the truth-value of a contingent statement we must wait for the realisation or failure of realisation of the event that contingent statement refers to before we can determine its truth-value. If Aristotle's argument is true it follows that the principle of bivalence is not true after all. If there are some statements that are not necessarily either true or false, then it cannot be true that all statements are necessarily either true or false. Aristotle concluded the argument by stating that:

> This is the case with regard to that which is not always existent or not always nonexistent. One of the two propositions in such instances must be true and the other false, but we cannot say determinately that this or that is false, but must leave the alternative undecided. One may indeed be more likely to be true than the other, but it cannot be either actually true or actually false. It is therefore plain that it is not necessary that of an affirmation and a denial one should be true and the other false. For in the case of that which exists potentially, but not actually, the rule which applies to that which exists actually does not hold good. The case is rather as we have indicated.[7]

Aristotle's argument can be stated as follows:

Premise 1: If all statements are necessarily either true or false, then statements on future events are necessarily either true or false.

Premise 2: If statements about the future are necessarily true or false, then everything that happens is fated to happen even before it happens.

Premise 3: If everything that happens is fated before it happens, then there is no chance and there is no free will. That is, everything that happens is predetermined. However, according to Aristotle, there is chance and there is freewill. Hence, statements about the future cannot be necessarily either true or false.

.: Not all statements are necessarily either true or false, as stated by the principle of bivalence.

If the principle of bivalence states that every statement is necessarily either true or false, and the statement "There will be a sea battle tomorrow" cannot be known to be true or false till after the occurrence or non-occurrence of the sea battle, the principle of bivalence fails in respect to some statements. Aristotle highlighted the problem, but did not proffer any solution to it.

Jan Lukasiewicz's Response to the problem of Contingent Events

Jan Lukasiewicz, popular in the field of logic for his three-valued propositional calculus, is the first recorded logician to create a system of many-valued logic. Lukasiewicz had to challenge the principle of non-contradiction[8] and the law of excluded middle to lay a foundation for his system of many-valued logic.

Jan Lukasiewicz on the Law of Non-Contradiction

> Today, like in the past, we believe that the principle of contradiction is the most reliable law of thought and being. Certainly only a fool could deny it. The validity of this law imposes itself on everyone with immediate evidence. It need not be founded, nor can

> it be. Aristotle taught us to believe this way. What is so surprising then, that nobody is concerned with something so clear, unquestionable and forever resolved?[9]

Lukasiewicz argued that though the law of non-contradiction seems convincing, it is not self-evident as Aristotle, Avicenna and some other philosophers claimed. Leibniz also argued that, "Everybody; even barbarians must tactically assume the law of contradiction as part of innate knowledge."[10]

These philosophers assumed that the law of non-contradiction needs no demonstration. Aristotle argued that not only is it unnecessary to demonstrate the law; it is impossible to demonstrate it. Any attempt to demonstrate it will lead to begging the question, because only the law can be used to prove itself.[11] Lukasiewicz disagreed with these philosophers and concluded that the law is not indispensable to logic.

Lukasiewicz claimed that Aristotle gave more than one definition for the law of non-contradiction. He classified the definitions into three; the ontological definition, the logical definition and the psychological definition. The ontological definition states no object can have and not have the same property. According to Aristotle, "For the same (property) cannot together belong and not belong to the same (object) and in the same respect."[12]

The logical definition states that two contradictory statements cannot both be true. According to Aristotle, "So that this is the most certain of all opinions: contradictory sentences are not together true."[13] The psychological definition states that two beliefs corresponding to two contradictory statements cannot exist together and in the same mind. Aristotle argued:

> For it is impossible for someone to believe that the same is and is not, as some people think Heraclitus said, but it isn't necessary that someone believes what he says.[14]

Lukasiewicz stated that the three definitions can only refer to the same definition if they are synonymous. Two expressions are synonymous when they express the same meaning. Consider two sentences P and Q, if we replace the subject of P with that of Q and the meaning P expresses is still the same, we can assume that P and Q are synonymous. For example, “The spinster is in the room” and the “The unmarried lady is in the room” are assumed to be synonymous because we can replace “The spinster” with “The unmarried lady” without a change in the meaning of the sentence.

Two expressions are equivalent when they have the same logical content or when the truth-value of one can be proved from the other. When two sentences have the same truth-value in all interpretations of the sentences, they are equivalent. If P and Q are sentences, they are equivalent if and only if, a biconditional of P and Q: P↔Q, (↔ is the symbol for the biconditional in symbolic logic), is true in all its interpretations. Where P is true, Q will also be true and where P is false, Q will also be false.

Lukasiewicz argued that the three definitions could not be referring to the same law. He argued that two statements are synonymous when the subjects refer to the same object and the predicate is the same. For example, the statements “Aristotle was the creator of Logic” and “Stagyrite was the creator of logic” are synonymous because the subject refers to the same object, and the predicate is the same. “Aristotle” and “Stagyrite” refer to the same person. The statements “Aristotle was a pupil of Plato” and “Plato was a teacher of Aristotle” are equivalent because if “Aristotle was a pupil of Plato”, then it follows that “Plato was a teacher of Aristotle”. If “Plato was a teacher of Aristotle” then it follows that “Aristotle was a pupil of Plato.”

The two statements: “Plato was a teacher of Aristotle” and “Aristotle was a pupil of Plato” are not synonymous because their subjects refer to different objects. The two statements: “Aristotle was the creator of logic” and “Stagyrite was the creator of logic” are not equivalent because there is no mutual implication between the two. That is, if “Aristotle was the creator of Logic” it does not follow that “Stagyrite was the creator of logic” and if “Aristotle was the creator of Logic” it does not follow that “Stagyrite was the creator of logic.”

Based on these explanations of synonymity and equivalence, Lukasiewicz argued that the three definitions Aristotle gave for the law of non-contradiction are not synonymous; hence, cannot be said to define one and the same principle. The subject of the ontological definition refers to objects and properties, the subject of the logical definition to statements, and the subject of the psychological definition to beliefs. However, the ontological and logical definitions are equivalent because one follows from the other. That is, if the ontological definition of the principle of non-contradiction is true, then the logical definition is also true.

The ontological definition states that no object can at the same time possess a property and lack that property at the same time. The logical definition states that two statements, of which the first ascribes to an object exactly that property which the second denies it, cannot be true together. In the language of symbolic logic, if ~(P∧~P) is true, then it follows that (P∧~P) is false. The ontological definition states that ~(P∧~P), while the logical definition states that P and ~P cannot be true together. When ~(P∧~P) is true, it is also true that P and ~P cannot be true together; the ontological and logical definitions are equivalent.

Lukasiewicz, however, outrightly rejected the psychological definition on the basis that relations between statements are different from that between beliefs. Lukasiewicz argued that a belief can only be proven empirically. Beliefs cannot be treated like pure logical objects, since they are necessarily related to experience. That is, a belief is too subjective and should not be considered a serious aspect of logic. Lukasiewicz argued that it is probably possible for a person to hold two contradictory beliefs in the mind at the same time. He cited Hegel as an example, “Something is in movement ... because in one and the same ‘now’ it is here and not here, it simultaneously is and not is.”[15]

Lukasiewicz argued that either Hegel did not mean what he said, or he actually held two contradictory beliefs in his mind. If it is possible for two beliefs to exist in the same mind at the same time, then an aspect of the principle of non-contradiction is not certain, hence should not be a fundamental principle of logic.

For Lukasiewicz, if it is possible for two beliefs to exist in the same mind and at the same time, then the psychological definition of the principle of non-contradiction is not certain. Any uncertain definition cannot be a fundamental aspect of logic. Logic strives for certainty and avoids subjective and uncertain concepts and arguments to a large extent. Lukasiewicz's argument can be summarised as: only empirically can we know that two contradictory beliefs can exist in the same mind and at the same time, or cannot exist in the same mind and at the same time. Experience cannot provide a fundamental foundation for logic. He wrote: "In 1910 I published a book on the principle of contradiction in Aristotle's work, in which I strove to show that, that principle is not as self-evident as it is believed to be."[16]

Lukasiewicz still had the ontological and logical definitions to deal with. He gave some arguments to show that they are not indispensable to logic as argued by Aristotle. Lukasiewicz rejected Aristotle's arguments that the principles of bivalence and non-contradiction are self-evident; that any attempt to prove them will be begging the question; that they are ultimate principles.

According to Lukasiewicz; any ultimate principle must be able to serve as proof of other principles or laws. But this is not the case with the law of non-contradiction. The law cannot serve as a proof for any law of logic. All other laws and principles of logic can be relevant without the law. That is, the law is not indispensable to logic. The burden of proof lies on the proponents of traditional logic to prove the law of contradiction and its indispensability.

For Lukasiewicz, the meaning of the word "true" must be assumed to justify or 'prove' the truth of a statement. The proof of any statement requires that the justifying statement be independent of the statement being proved. Lukasiewicz argued that the law of identity, which states that "If a statement is true, then the statement is true", needs the meaning of the word "true" in logic to prove the truth of the law. In order to be accepted as true, it is necessary to prove a principle of logic as true independent of that principle itself. Lukasiewicz argued thus: "Every other a priori basic law, even the principle of contradiction, must be derived from previously demonstrated principles outside it."[17]

If a principle or law is self-evident, then every rational human should be able to see its truth; however, not everyone accepts the law of non-contradiction. Lukasiewicz argued that the only possible justification for the law of non-contradiction is based on the definition of objects. An object is *what does not possess contradictory properties.* Aristotle did not give any argument to show that objects, indeed, do not possess contradictory properties. We cannot just assume that it is true that objects do not have contradictory properties; we have to give arguments for the claim. There may be objects that possess contradictory properties, for example 'the square built by rule and compasses and identical as regards the surface area to the circle of a radius of 1', transfinite numbers[18] and Russell's antinomies.[19]

Lukasiewicz also rejected Aristotle's argument that the law of non-contradiction is valid based on evidence. Lukasiewicz argued that evidence refers to a mental state. Mental states can only be investigated empirically. Empirical investigations are sometimes based on individual observations. This may lead to subjectivism and scepticism.[20] That is, if A considers a statement true by evidence, it is true for A. If B considers the same statement as not true by evidence, then it is not true for B. The implication of this is that some statements will be evident for some people and not for others. Lukasiewicz's argument is that except everyone accepts a principle or law as evident based on a valid proof, we cannot just declare it as self-evident. In logic, a principle or law cannot just be evident; it should have a valid proof independent of people's opinion.

According to Lukasiewicz, the law of identity and principle of double negation are simpler and can still be called 'self-evident'. The truth of both the law and principle can be demonstrated if necessary. However, the law of non-contradiction is not self-evident as it has "objects" in its definition. An object may contain contradictories, unless we prove otherwise. However, this is only a formal proof and not a concrete proof. According to Lukasiewicz, a principle or law that serves as a foundation to logic must have more than a formal proof; it requires a concrete proof.[21]

The type of proof that the law of non-contradiction requires is a proof that shows that, everything that is an object does not contain

contradictories. We cannot just assume that objects are devoid of contradictories, we have to show that they are actually so in order to use the principle as a proof for a principle or law that can be accorded some logical worth. However, Lukasiewicz argued that this type of proof is not possible.[22]

For Lukasiewicz, though the law of non-contradiction may be useful, it can only be accepted as probable. Any principle to be will be fundamental to logic, must go beyond just being probable. It must have a proof. To call it an indemonstrable law is to doubt its credibility as a useful tool in logic. Lacking demonstrable evidence, it has little or no logical worth. Logic requires laws and principles that have objective and well-demonstrated proofs.

Reactions to Aristotle

From Aristotle's essay on future contingents, we cannot infer any intention of laying a foundation for many-valued logics; he only intended to preserve the concept of freewill. Aristotle gave no argument for fatalism. He probably thought that to say that the truth-value of statements referring to future events is restricted to truth and falsity, is to pretend that the future is already determined and this would have unacceptable consequences on human life, especially with regards to ethics.

One observation on Aristotle's argument is that Aristotle failed to argue for why fatalism is unacceptable and free will is acceptable. Aristotle seems to accept that it is self evident that fatalism is unacceptable and free will acceptable. Even if to determine the truth-value of a statement of a future event prior to its occurrence will indeed lead to fatalism, there is the need to argue that fatalism is unacceptable (false).

Another observation is Aristotle seems to misunderstand the principle of bivalence. The principle does not assert that all statements can be known to be true or false, but that all statements are necessarily true or false. Aristotle confused an epistemic problem for a logical problem. That we cannot know whether a statement about future events is true or false is epistemic. That is, it shows one of the limits of human

knowledge. That we cannot know if a statement is necessarily either true or false is a claim about the logical property of bivalence.

Furthermore, if I say that "It is true that there will be a sea battle tomorrow" today and tomorrow there is actually a sea battle, it does not follow that the event has been predetermined. There is nothing that shows that the event has been predetermined because I said it will happen and it actually happened. I could have guessed right anyway.

Reactions to Lukasiewicz

There is no evidence that Aristotle gave three different definitions for the same law of contradiction as suggested by Lukasiewicz. Aristotle gave different illustrations to explain the law. It seems Lukasiewicz merely adapted one of Aristotle's illustrations and consequence of the law of non-contradiction and turned it into a psychological definition of the law. Aristotle could have inferred a faulty consequence or given a wrong interpretation of the law, but it does not follow that the wrong interpretation or its assumed consequence is actually a definition of the law.

Lukasiewicz also equivocated on the words "evidence" and "evident." "Evidence" is an information or testimony, whether oral or written, presented to indicate the truth of an issue or situation. While "evident" means that something is plain, clear or obvious. Lukasiewicz argued that if we accept that the law of contradiction is valid based on evidence it will lead to subjectivism. Lukasiewicz in the same argument stated that an argument cannot be evident based on people's opinion. According to Lukasiewicz, "Evidence is not a truth criterion, since even false judgements may turn out to be evident..."[23]

However, Aristotle's argument is that the law of contradiction is clear and obvious and needs no further proof. Lukasiewicz probably assumed that we need an evidence to know that something is evident.

We may not be able to establish that a statement is true or that the statement is false, but this does not affect the truth of the assumption that the statement is either true or false. For example, we may not be able to establish that it is true or that it is false that "There will be a

sea battle tomorrow," but we can rightly assume that it is either true or false that "There will be a sea battle tomorrow."

Lesniewski rejected Lukasiewicz's arguments against Aristotle on the law of non- contradiction. Lesniewski, in his book, An Attempt at a Proof of the Principle of Contradiction in Aristotle, responded to some of the arguments Lukasiewicz raised against Aristotle on the principle of contradiction. Lesniewski paid particular attention to the ontological definition of the law of contradiction, since it served as the main source of Lukasiewicz's rejection of the law.[24]

Lesniewski agreed that the law should be proved, but he disagreed with Lukasiewicz on his argument that the law cannot be proved. Lesniewski's arguments focused on Lukasiewicz's claims that first; the ontological definition of the law of contradiction cannot be proved, second; that there is a difference between a formal proof and a concrete proof and third; that there are contradictory objects in formal constructions and probably in reality too. Lesniewski used semantic and ontological arguments to prove his position that the law of contradiction is true.[25]

Lesniewski argued that the ontological formulation of the law of contradiction is true and can be proved. Lesniewski defined a sentence with symbolic function as a sentence that symbolises properties, not objects. He defined a connotative statement as a statement that signifies something specific. For example, "All men are mortal" has a symbolic function because it has the property to symbolise an object "man." It is also connotative because it signifies a property in specific. Connotative expressions are those that can be defined by a class of common attributes, for example, "man" or "tree." However, 'object' is non-connotative because it symbolises anything and cannot be defined by a class of common attributes, or else will fall into infinite regress.[26]

According to Lesniewski, a sentence about 'objects' in general is not connotative because it signifies no specific object. Furthermore, no object can be said to be a general object. If we call an object a "general object", that is, an object that represents all objects, it is possible to find one object that will not fit into the specifications of the "general object" in some context in which it is used. If it is

possible to have at least one object that will not fit into the specifications of the "general object", then it is not correct to use 'objects' generally.[27] Lesniewski stated that a sentence is true when it has a symbolic function and is connotative, otherwise false. A sentence that talks about a "general object" is not connotative because it refers to no object in specific. Hence, it is false.[28]

A Construction of Lesniewski's Argument

(1) A general object PK is an object that corresponds to individual objects P'1, P'2, P'3 ... P'n. This implies that

(2) Every individual object P'1, P'2, P'3 ... P'n has the property pk. But,

(3) For every general object Pk, there is an individual object P'1 or P'2 or P'3 or P'4 or P'n, which lacks the property pk. (This is empirically true). But,

(4) If some individual object pk lacks the property of possessing the property pk, it would be a contradictory object, since it possesses (from premise 2) and does not possess (from premise 3) the property k. From (3) it follows that

(5) The property of possessing the property pk is not common to all the individual objects $P_1, P_2, P_3 \ldots P_n$.

(6) The general object possesses the property of not possessing the property pk. (This follows from (1) since the general and the individual objects have a relationship of correspondence).

Therefore, from (1) and (6), it follows that

(7) A general object PK possesses and does not possess the property pk.

So, the assumption that a general property is an object corresponding to individual objects leads to a contradiction. Hence, the claim that there are general objects is false. No objects are general.[29]

Lesniewski argued that Lukasiewicz's interpretation of the ontological formulation of the law of contradiction uses "general object." Hence, lukasiewicz's understanding of the law is not true and is not synonymous with the ontological formulation of the law.[30] For

Lesniewski, two sentences are synonymous when the subjects of both sentences connote or signify exactly the same property. Otherwise, they are not synonymous.[31] For example, for Lesniewski, "Aristotle was the creator of logic" and "The Stagyrite was the creator of logic" are not synonymous because the subject of the former sentence connotes the property of the name "Aristotle," while "The Stagyrite" does not. They denote the same object, but to be synonymous they should also connote the same property.

On Lesniewski's View

Lukasiewicz's interpretation of the ontological formulation of the law of the contradiction actually connotes nothing, because Lukasiewicz assumes a 'general property'. A sentence that refers to a 'general property' is false because it connotes nothing in particular. That is, for Lesniewski "All objects..." is not synonymous with "An object..." There is no such object as "All objects" but "An object" in particular exists.

There is a huge difference between Lesniewski's ontology and Lukasiewicz's. Lesniewski assumes that only individual objects are real. General objects do not exist.[32] For example, "Wooden iron" and "Round Square" do have meanings but wooden iron and round squares in general do not exist. While Lukasiewicz has a broader ontology; if it is possible for a contradictory object, even if it is general, to exist in some human mind then it probably exists in reality or experience.

However, for Lesniewski anything that does not exist in experience is non-existent. Any sentence that connotes a non existing object is false, because it connotes nothing in particular. Lesniewski argued that any object that really exists connotes a property. The object cannot have and not have that property at the same time and in the same context. The ontological formulation of the principle of contradiction refers to an object that exists and not to a non-existing object.

According to Lesniewski, the proof needed for the ontological formulation of the law of contradiction is: If P is something and is not nothing, then P is a non-contradictory object.[33] Lesniewski also argued

that there is no difference between a formal proof and a concrete proof as suggested by Lukasiewicz. If semantics and the accepted conventions show that the law of contradiction is true, then we do not need any other proof.[34] If the law is true we only need to prove that it is true. In the case of the law of contradiction, it is true because it is a sentence that has a symbolic function and is connotative.

For Lukasiewicz the ontological definition of the principle of contradiction states that "Every object cannot have and not have the same property at the same time" is synonymous with the conditional form "If A is an object then it cannot have and not have the same property at the same time". For Lukasiewicz a general judgement presents a link between two judgements. The two statements: "Every object cannot have and not have the same property at the same time" and "If A is an object then it cannot have and not have the same property at the same time" are synonymous, according to Lukasiewicz.

For Lesniewski, however, the two statements are not synonymous as long as their objects are non-connotative. Lukasiewicz appealed to the traditional definition of object, while Lesniewski appealed to its semantic and linguistic use. For Lesniewski, as far as linguistics is concerned the statement "Every object cannot have and not have the same property at the same time" is true, hence the ontological definition of the principle of contradiction is true. However, for Lukasiewicz, unless we can present a concrete proof that objects are non-contradictory, we should not claim that the definition is true.

The differences between Lukasiewicz and Lesniewski lie in their diverse interpretations on synonymy and ontology. Lesniewski failed to argue for why we should accept only things that exist in experience in our ontology. If it is possible to argue that there exist more objects in our ontology than Lesniewski allows, then his defence will collapse. He needs to show that indeed only the world of experience exists and is acceptable in our ontology. Lesniewski is an extreme nominalist and he needs to defend his extreme nominalism.

Conclusion

The principle of bivalence asserts that something is true **or** false, not that something is true and something is false. We may not be able to establish, for every statement, that it is true or false. To assume that the principle of bivalence asserts that we can establish every statement to be true or false is to confuse knowledge with truth. The fact that we do not have the knowledge of the value of a statement does not mean that it cannot have a value. Establishing our inability to know whether a statement is true, or that it is false, is not the same thing as establishing that it is either true or false. To establish that a statement is true or false, we first assume that it is either true or that it is false.

Aristotle actually challenged the truth of the principle of bivalence. However, although there are difficulties in determining whether some declarative statements are true or false, this does not imply that such statements are not capable of being true or false.

The argument that the principle of bivalence is limited or false, as argued by Lukasiewicz and Aristotle, is not right. The principle of bivalence is right to assert that all statements are necessarily either true or false. But it is wrong to assume that the principle of bivalence also means that we can know, at all times and immediately, when a statement is true or false. The principle of bivalence is the logical core of classical logic and should not be jettisoned. There is nothing wrong with creating systems that can handle statements that we cannot establish as true or false. But this is not equivalent to a rejection of the principle of bivalence.

Endnotes

1. These laws date back to the pre-Socratic period. However, Aristotle is regarded as the founder of the laws, in western philosophy.
2. Quine W.V.O. *Philosophy of Logic*. (New Jersey: Prentice-Hall, 1970), p.83.
3. J. C. Ackrill. Aristotle: *Aristotle Categories and De Interpretatione.* (Oxford: Clarendon Press, 1966), p. 37.

4. Diodorus of Cronus is a Greek philosopher of the Megarian school. He lived in the second half of the 4th Century B.C. Most of his works are in fragmentary information. His argument on possibility, also known as the master argument, was summarised by Epictetus.
5. Ackrill. Aristotle: *Aristotle Categories & De Interpretatione*, p.54.
6. Ibid., p.55
7. Ibid., p.63.
8. Note that the use of law of 'non-contradiction' was not used in the time of Aristotle and Lukasiewicz's writings; it was called the principle of contradiction. However, in recent times the principle has been known as the law of non-contradiction, as against principle of contradiction.
9. *Ludwik Borkowski*. Selected works: Lukasiewicz J. (Oxford: North-Holland, 1970), p.47.
10. Kneale W. and Kneale M. *The Development of Logic*), p.65.
11. Ackrill. Aristotle: *Aristotle Categories & De Interpretatione.,* p.52.
12. Ibid., p.55.
13. Ibid.
14. Ibid., p.57.
15. Lukasiewicz, J. *On the Principle of Contradiction in Aristotle,* Translated by V. Wedin, *Review of Metaphysics* (1971), Vol.1, No.3, p.485.
16. An excerpt from a farewell lecture delivered by Jan Lukasiewicz at the Warsaw University on the 7th March, 1918.
17. Lukasiewicz, *On the Principle of Contradiction in Aristotle*, p.487.
18. Transfinite numbers are cardinal or ordinal numbers that are larger than all finite numbers, yet not necessarily absolutely infinite. For example, aleph-null and omega.
19. Russell's paradoxes or antinomies were discovered by Bertrand Russell in 1901 to show that Frege's naive set theory leads to a contradiction. See Bertrand Russell, *Principles of Mathematics*, (Cambridge: Cambridge University. Press, 1903).

20. Lukasiewicz J. *On the Principle of Contradiction in Aristotle,* p.498.
21. Ibid., p.511.
22. Ibid.
23. Ibid.
24. Luschei E. *The Logical Systems of Lesniewski.* (Oxford: North-Holland, 1962), pp.119 – 120.
25. Ibid.
26. Ibid., p.120.
27. Ibid., p.210.
28. Ibid., pp.210 – 211.
29. Ibid., pp. 103 – 105.
30. Ibid., p.98.
31. Ibid., p.219.
32. Ibid., p.222.
33. Ibid., p.213.
34. Ibid., p.233.